SSLC - MATHEMATICS PROGRESSION

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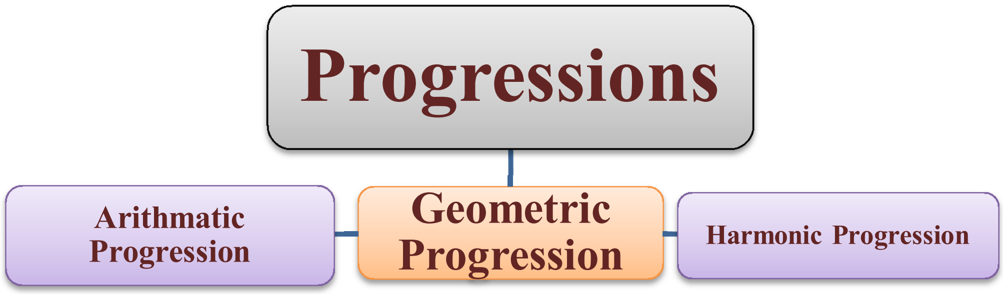
Chapter-2



Arithmatic Progression:

General form: $a, (a + d), (a + 2d), (a + 3d) - - - \quad a + (n - 1)d$

'n' th term of Arithmatic Progression: $T_n = a + (n - 1)d$

a – First term, n – Number of terms, d – Common difference

$$T_{n+1} = T_n + d$$

$$T_{n-1} = T_n - d$$

$$d = \frac{T_p - T_q}{p - q}$$

$$d = \frac{T_n - a}{n - 1} \quad [T_p = T_n] \text{ and } [T_q = 1]$$

Sum of first 'n' terms of an A.P.:

$$S_n = \frac{n}{2}[2a + (n - 1d]$$

a – First term, n – Number of terms, d – Common difference

Sum of first 'n' natural numbers:

$$\sum_1^n n = \frac{n(n+1)}{2}$$

$$S_n = \frac{n}{2}[a + T_n]$$

Geometric Progression:

General form: $a, ar, ar^2, ar^3 - \quad - \quad - ar^{n-1}$

'n' th term of Arithmatic Progression:

$$T_n = ar^{n-1}$$

$$T_{n+1} = T_n \times r$$

$$T_{n-1} = \frac{T_n}{r}$$

Sum of first 'n' terms of an G.P:

$$S_n = a\left(\frac{r^n-1}{(r-1)}\right) \quad r > 1$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \quad r < 1$$

$$S_{2n} : S_n = 1+r^n : 1$$

Sum of an infinite Geometric Series:

$$S_\infty = \frac{a}{1-r}$$

Harmonic Progression:

General form of H.P.: $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d} - - - \frac{1}{a+(n-1)d}$

A sequence in which, the reciprocals of the terms form an artithmetic progression is called Harmonic progression.

'n' th term of H.P. $T_n = \frac{1}{a+(n-1)d}$

Mean

- **Arithmetic Mean**: $A = \frac{a+b}{2}$
- **Geometric Mean**: $G = \sqrt{ab}$
- **Harmonic Mean**: $H = \frac{2ab}{a+b}$

ILLUSTRATIVE EXAMPLES

1: Find the first three terms of the sequence whose n^{th} term is $2n + 3$

Sol: $T_n = 2n + 3$

$T_1 = 2x1 + 3 = 2 + 3 = 5$

$T_2 = 2x2 + 3 = 4 + 3 = 7$

$T_3 = 2x3 + 3 = 6 + 3 = 9$

∴ first three terms **5, 7, 9**

2: Find the first four terms of the sequence whose n^{th} term is $\frac{n^2}{n+1}$

Sol: $T_n = \frac{n^2}{n+1}$

$T_1 = \frac{1^2}{1+1} = \frac{1}{2}$
$T_2 = \frac{2^2}{2+1} = \frac{4}{3}$
$T_3 = \frac{3^2}{3+1} = \frac{9}{4}$
$T_4 = \frac{4^2}{4+1} = \frac{16}{5}$
∴ The first four terms $\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}$

3: Find the 20^{th} term of the sequence if $T_n = 3n - 10$.
Sol: $T_n = 3n - 10$
$T_{20} = 3 \times 20 - 10$
$T_{20} = 60 - 10$
$T_{20} = 50$

4:If $T_n = 5n + 2$ find T_{n+1} .
Sol: $T_{n+1} = 5(n+1) + 2$
$T_{n+1} = 5n+5 + 2$
$T_{n+1} = 5n+7$

5:If $T_n = n^3 – 1$ find the value of 'n' so that $T_n = 26$.
$T_n = n^3 – 1$
$26 = n^3 – 1$
$26 + 1 = n^3$
$n^3 = 27$
$n = 3$

Exercise 2.1

1. Which of the following form a sequence?
   (i) 4, 11, 18, 25, …. sequence
   (ii) 43, 32, 21, 10….. sequence
   (iii) 27, 19, 40, 70,…… Not a sequence
   (iv) 7, 21, 63,189, ….. sequence
2. Write the next two terms of the following sequences.
   (i) 13, 15, 17, -, -, Ans: 19 , 21
   (ii) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, -, - Ans: $\frac{5}{6}, \frac{6}{7}$
   (iii) 1, 0.1, 0.01, -, - Ans: 0.001, 0.0001
   (iv) 6, 12, 24, -, - Ans: 48. 96
3. If $T_n = 5 - 4n$ find the first three terms.
   $T_n = 5 - 4n$
   $T_1 = 5 - 4 \times 1 = 5 – 4 = 1$
   $T_2 = 5 - 4 \times 2 = 5 – 8 = -3$
   $T_3 = 5 – 4 \times 3 = 5 – 12 = - 7$
4. If $T_n = 2n^2 + 5$, find (i) T_3 and (ii) T_{10}

(i) $T_3 = 2 \times 3^2 + 5$
$T_3 = 2 \times 9 + 5$
$T_3 = 18 + 5$
$T_3 = 23$

(ii) $T_{10} = 2 \times 10^2 + 5$
$T_{10} = 2 \times 100 + 5$
$T_{10} = 200 + 5$
$T_{10} = 205$

5. If $T_n = n^2 - 1$,find (i) T_{n-1} and (ii) T_{n+1} .

(i) $T_{n-1} = (n-1)^2 - 1$

$T_{n-1} = n^2 - 2n + 1 - 1$

$T_{n-1} = n^2 - 2n$

(ii) $T_{n+1} = (n+1)^2 - 1$

$T_{n+1} = n^2 + 2n + 1 - 1$

$T_{n+1} = n^2 + 2n$

6. If $T_n = n^2 + 4$ an $T_n = 200$ find the value of 'n'.

$T_n = n^2 + 4$

$200 = n^2 + 4$

$n^2 + 4 = 200$

$n^2 = 200 - 4$

$n^2 = 196$

$n^2 = 14$

ILLUSTRATIVE EXAMPLES

1: Check whether 13,19,25,31,…. form an A.P.

Sol: $T_1 = 13$, $T_2 = 19$, $T_3 = 25$, $T_4 = 31$

$d = T_2 - T_1 = 19 - 13 = 6$

$d = T_3 - T_2 = 25 - 19 = 6$

$d = T_4 - T_3 = 31 - 25 = 6$

Since there is a common difference, the given sequence is an A.P.

2: If a = 3 and c.d = 4, find the A.P .

Sol: $T_1 = 3$,

$T_2 = a + d = 3 + 4 = 7$

$T_3 = 7 + 4 = 11$

$T_4 = 11 + 4 = 15$

∴ The A.P is 3, 7, 11, 15…..

3: In the A.P 12, 19, 26,...... find T_n and hence find T_{15}.

Sol: a = 12, d = 7,

$T_n = a + (n - 1)d$

$T_n = 12 + (n - 1)7$

$T_n = 12 + 7n - 7$

$T_n = 7n + 5$

$T_{15} = 7x15 + 5$

$T_{15} = 105 + 5$

$T_{15} = 110$

4: Find the number of terms in the finite A.P 7, 13, 19, ...,.

Sol: a = 7, d= 6, $T_n = 151$

$T_n = a + (n - 1)d$

$151 = 7 + (n - 1)6$

$151 = 7 + 6n - 6$

$151 = 1 + 6n$

$6n = 151 - 1$

$6n = 150$

$n = 25$

5: The 8^{th} term of an A.P is 17 and the 19^{th} term is 39. Find the 25^{th} term.

Sol: $T_8 = 17$, $T_{19} = 39$, $T_{25} = ?$,

$d = \frac{T_p - T_q}{p-q}$

$d = \frac{T_{19} - T_8}{19-8}$

$d = \frac{39-17}{11}$

$d = \frac{22}{11}$

d = 2

$T_p = T_q + (p-q)d$

$T_{25} = T_{19} + (25 - 19)d$

$T_{25} = 39 + 6x2$

$T_{25} = 39 + 12$

$T_{25} = 51$

6: Determine the A.P. whose 4^{th} term is 17 and the 10^{th} term exceeds the 7^{th} term by 12

$T_4 = 17$, $T_{10} = T_7 + 12$

a + 9d = a + 6d + 12

9d – 6d = 12

3d = 12

d = 4

$T_4 = 17$

a + 3d = 17

a + 3x4 = 17

a + 12 = 17

a = 17 – 12

a = 5

∴ The A.P is 5, 9, 13, 17 . . . .

7: In an A.P, 7 times the 7^{th} term is equal to 11 times the 11^{th} term. Find the 18^{th} term of the A.P.

$7T_7 = 11T_{11}$

7(a + 6d) = 11(a + 10d)

7a + 42d = 11a + 110d

7a – 11a = 110d – 42d

-4a = 68d

-a = 17d

a = - 17d

$T_{18} = a + 17d$

$T_{18} = -17d + 17d$

$T_{18} = 0$

Exercise 2.2

1. Write the next four terms of the following A.P.

(i) 0, -3, -6, . . . (ii) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$ . . . (iii) a + b, a – b, a – 3b, . . .

(i) 0, -3, -6, -9, -12, -15,-18

(ii) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \frac{7}{6}$

(iii) a + b, a – b, a – 3b, a – 5b, a – 7b, a – 9b, a – 11b

2. Find the sequence if.

(i) $T_n = 2n - 1$ (ii) $T_n = 5n + 1$

(i) $T_1 = 2x1 - 1$ $T_2 = 2x2 - 1$ $T_3 = 2x3 - 1$

$T_1 = 2 - 1$ $T_2 = 4 - 1$ $T_3 = 6 - 1$

$T_1 = 1$ $T_2 = 3$ $T_3 = 5$

∴ The sequence is : 1, 3, 5, 7, 9. . . .

(ii) $T_1 = 5x1 + 1$ $T_2 = 5x2 + 1$ $T_3 = 5x3 + 1$

$T_1 = 5 + 1$ $T_2 = 10 + 1$ $T_3 = 15 + 1$

$T_1 = 6$ $T_2 = 11$ $T_3 = 16$

∴ The sequence is : 6, 11, 16, 21, 26 . . . .

3. In an A.P,

(i) If a = -7, d = 5 find T_{12}.

(ii) If a = -1, d = -3 find T_{50}.

(iii) If a = 12, d = 4, T_n = 76 find 'n'.

(iv) If d = -2, T_{22} = -39 find 'a'.

(v) If a = 13, T_{15} = 55 find 'd' .

(i) If a = -7, d = 5 find T_{12}.

$T_n = a + (n – 1)d$

$T_{12} = -7 + (12 – 1)5$

$T_{12} = -7 + 11x5$

$T_{12} = -7 + 55$

$T_{12} = 48$

(ii) If a = -1, d = -3 find T_{50}.

$T_n = a + (n – 1)d$

$T_{50} = -1 + (50 – 1)-3$

$T_{50} = -1 + 49x-3$

$T_{50} = -1 + -147$

$T_{50} = -148$

(iii) If a = 12, d = 4, T_n = 76 find 'n'

$a + (n – 1)d = T_n$

$12 + (n – 1)4 = 76$

$12 + 4n – 4 = 76$

$4n + 8 = 76$

$4n = 76 - 8$

$4n = 68$

$n = 17$

(iv) If d = -2, T_{22} = -39 find 'a' .

$a + (n – 1)d = T_n$

$a + (22 – 1)-2 = -39$

$a + (21)x-2 = -39$

$a - 42 = -39$

$a = -39 +42$

$a = 3$

(v) If a = 13, T_{15} = 55 find 'd'

$a + (n – 1)d = T_n$

$13 + (15 – 1)d = 55$

$13 + 14d = 55$

$14d = 55 - 13$

$14d = 42$

$d = 3$

4. Find the number of terms in the A.P,100, 96, 92, . . . , 12.

$a = 100, d = T_2 - T_1$ $96 – 100 = -4$, $T_n = 12$

$a + (n – 1)d = T_n$

$100 + (n – 1)-4 = T_n$

$100 -4n + 4 = 12$

$-4n + 104 = 12$

$-4n = 12 - 104$
$-4n = -92$
$n = 23$

5. The angles of a triangle are in A.P. If the smallest angle is 50°, find the other two angles.

a, a + d, a + 2d are the 3 angles of a triangle. The smallest angle $a = 50^0$
$a + (a+d) + (a+2d) = 180^0$ [∵Sum of 3 angles of a triangle is 180^0]
$50^0 + (50^0 + d) + (50^0 + 2d) = 180^0$
$150^0 + 3d = 180^0$
$3d = 180^0 – 150^0 = 30^0$
$d = 10^0$
∴ 3 angles of a triangle $50^0, 50^0 + 10^0, 50^0 + 2x10^0 = 500, 60^0, 70^0$

6. An A.P consists of 50 terms of which 3^{rd} term is 12 and last term is 106. Find the 29^{th} term 50.

$n = 50, T_3 = 12, T_{50} = 106, T_{29} = ?$
$d = \frac{T_p - T_q}{p - q}$
$d = \frac{T_{50} - T_3}{50 - 3}$
$d = \frac{106 - 12}{47}$
$d = \frac{94}{47} = 2 \Rightarrow d = 2$
$T_{29} = T_3 + 26d$ [$\because T_p = T_q + (p-q)d$]
$T_{29} = 12 + 26x2$
$T_{29} = 12 + 52$
$T_{29} = 64$

7. The sum of 4^{th} and 8^{th} terms of an A.P is 24 and the sum of 6^{th} and 10^{th} terms of the same A.P is 44. Find the first three terms.

$T_4 + T_8 = 24, T_6 + T_{10} = 44$
$T_4 + T_8 = 24$
$\Rightarrow a + (4-1)d + a + (8-1)d = 24$
$\Rightarrow a + 3d + a + 7d = 24$
$\Rightarrow 2a + 10d = 24$
$\Rightarrow a + 5d = 12$ ------(1)
$T_6 + T_{10} = 44$
$\Rightarrow a + (6-1)d + a + (10-1)d = 44$
$\Rightarrow a + 5d + a + 9d = 44$
$\Rightarrow 2a + 14d = 44$
$\Rightarrow a + 7d = 22$ -------(2)
(1) - (2)
$a + 5d = 12$
$a + 7d = 22$
$-2d = -10$

$d = 5$
Substitute $d = 5$ in Eqn (1),
$a + 5x5 = 12 \Rightarrow a + 25 = 12 \Rightarrow a = 12 – 25 = -13$
∴ The first three terms are : a , a+d, a + 2d ⇒ -13, -13+5, -13 + 10 ⇒ -13 , -8, -3

8. The ratio of 7^{th} to 3^{rd} term of an A.P is 12 : 5. Find the ratio of 13^{th} to 4^{th} term.

$\frac{T_7}{T_3} = \frac{12}{5} \Rightarrow 5T_7 = 12T_3$
$\Rightarrow 5[a + (7-1)d] = 12[a + (3-1)d]$
$\Rightarrow 5a + 30d = 12a + 24d$

$\Rightarrow$ 30d -24d= 12a – 5a

$\Rightarrow$ 6d = 7a

$\Rightarrow a = \frac{6d}{7}$

$\frac{T_{13}}{T_4} = \frac{a+(13-1)d}{a+(4-1)d}$

$\frac{T_{13}}{T_4} = \frac{\frac{6d}{7}+12d}{\frac{6d}{7}+3d}$

$\frac{T_{13}}{T_4} = \frac{\frac{6d+84d}{7}}{\frac{6d+21d}{7}}$

$\frac{T_{13}}{T_4} = \frac{10}{3}$

$T_{13} : T_4 = 10 : 3$

9. A company employed 400 persons in the year 2001 and each year increased by 35 persons. In which year the number of employees in the company will be 785?

This is in A.P.,

$\therefore$ a = 400 d = 35, T_n = 785

a + (n-1)d = T_n

400 + (n-1)35 = 785

400 + 35n-35 = 785

35n = 785 - 365

35n = 420

$n = \frac{420}{35}$

n = 12

$\therefore$ After 12 years the number of employees in the company will be 785.

$\Rightarrow$ 2001 + 12 = 2013th year

10. If the pth term of an A.P is q and the qth term is p, prove that the nth term equal to (p + q – n).

$d = \frac{T_p - T_q}{p-q}$

$d = \frac{q-p}{p-q}$

d = -1

$T_n = T_p + (n - p)d$

T_n = q + (n –p)x-1

T_n = q -n + p

$T_n = p + q - n$

11. Find four numbers in A.P such that the sum of 2nd and 3rd terms is 22 and the product of 1st and 4th terms is 85.

$T_2 + T_3 = 22$, $T_1 x\ T_4 = 85$

$T_2 + T_2 + d = 22$

a + (2-1)d + a+(2-1)d+ d = 22

a + d + a+d+ d = 22

2a + 3d = 22

a + 3d = 22 – a ------------(1)

$T_1 x\ T_4 = 85$

a[(a + (4-1)d] = 85

a[(a + 3d] = 85

a[22 - a] = 85 [Substituting (1)]

$22a - a^2 = 85$

$-a^2 + 22a - 85 = 0$

$a^2 - 22a + 85 = 0$

$a^2 - 17a - 5a + 85 = 0$

$a(a - 17) - 5(a - 17) = 0$

$(a - 17)(a - 5 = 0$

$a = 17$ Or $a = 5$

When Sustituting in (1)

$17 + 3d = 22 - 17 \Rightarrow 17 + 3d = 5 \Rightarrow 3d = 5 - 17 \Rightarrow d = \frac{-12}{3} \Rightarrow d = -4$

Or

$5 + 3d = 22 - 5 \Rightarrow 5 + 3d = 17 \Rightarrow 3d = 17 - 5 \Rightarrow d = \frac{12}{3} \Rightarrow d = 4$

∴The four terms are: 17 , 13, 9, 5 Or 5, 9, 13, 17

ILLUSTRAYIVE EXAMPLES

1:If $T_n = 2n - 1$ find S_3.

Sol:: $S_3 = T_1 + T_2 + T_3$

$T_n = 2n - 1$

$T_1 = 2x1 - 1 = 2 - 1 = 1$

$T_2 = 2x2 - 1 = 4 - 1 = 3$

$T_3 = 2x\,3 - 1 = 6 - 1 = 5$

$\therefore S_3 = T_1 + T_2 + T_3 = 1 + 3 + 5 = 9$

2:IF $T_n = n^2 + 1$ find S_2

Sol:$S_2 = T_1 + T_2$

$T_n = n^2 + 1$

$T_1 = 1^2 + 1 = 1 + 1 = 2$

$T_2 = 2^2 + 1 = 4+1 = 5$

$\therefore S_2 = T_1 + T_2 = 2 + 5 = 7$

ILLUSTRAYIVE EXAMPLES

1:If T_n=5n–2 find S_4

Sol:: $S_4 = T_1 + T_2 + T_3 + T_4$

$T_n = 5n - 2$

$T_1 = 5x1 - 2 = 5 - 2 = 3$

$T_2 = 5x2 - 2 = 10 - 2 = 8$

$T_3 = 5x3 - 2 = 15 - 2 = 13$

$T_4 = 5x4 - 2 = 20 - 2 = 18$

$\therefore S_4 = T_1 + T_2 + T_3 + T_4 = 3 + 8 + 13 + 18 = 42$

2: Find the sum of first 20 terms of the series 1 + 2 + 3 + ...

Sol:$\sum n = \frac{n(n+1)}{2}$ [Sum of first n' natural numbers]

$\sum 20 = \frac{20(20+1)}{2}$

$\sum 20 = \frac{20x21}{2}$

$\sum 20 = 10x21$

$\sum 20 = 210$

3: Find the sum of the first 15 terms of the A.P 5, 8, 11, 14, ...

Sol:$S_n = \frac{n}{2}[2a + (n-1)d]$

n = 15, a = 5, d = 3

$S_{15} = \frac{15}{2}[2x5 + (15-1)3]$

$S_{15} = \frac{15}{2}[10 + 14x3]$

$S_{15} = \frac{15}{2}[10 + 42$

$S_{15} = \frac{15}{2}$ x 52

S_{15} = 15x 26

S_{15} = 390

4: Find the sum of all natural numbers between 200 and 300 which are exactly divisible by 6.

Sol:The sum of all natural numbers between 200and 300 which are exactly divisible by 6,

204 + 210 + 216+ . . . . .+ 294

a = 204, d = 6, T_n = 294

T_n = a + (n – 1)d

294 = 204 + (n – 1)6

294 – 204 = 6n – 6

90 + 6 = 6n

6n = 96

n = 16

$S_n = \frac{n}{2}[a + T_n]$

$S_{16} = \frac{16}{2}[204 + 294]$

$S_{16} = 8[498]$

$S_{16} = 3984$

5: Ramesh wants to buy a cell phone. He can buy it by paying Rs 15,000 cash or by making 12 monthly instalments as Rs 1800 in the 1st month, Rs 1750 in 2nd month Rs 1700 in 3rd month and so on. If he pays the money in instalments. find,

(i) total amount paid in 12 instalments 12

(ii) how much extra he has to pay over and above the cost price.

$S_n = \frac{n}{2}[2a + (n-1)d$

n = 12, a = 1800, d = -50

$S_{12} = \frac{12}{2}[2\text{x}1800 + (12-1) - 50]$

$S_{12} = 6[3600 + 11\text{x} - 50]$

$S_{12} = 6[3600 - 550]$

S_{12} = 6x3050

S_{12} = 18,300

∴ Total amount paid in 12 instalments = Rs 18300.

Extra amount paid = 18300-1500 = Rs 3,300

6: Find three positive integers in A.P such that their sum is 24 and their product is 480.

Let the three terms be: a – d, a, a + d,

a – d + a + a + d = 24

3a = 24

∴a = 8

(a – d) x a x (a + d) = 480

(8 – d)x8x(8 + d) = 480

(8 – d)(8 + d) = 60

$8^2 - d^2 = 60$

$64 - d^2 = 60$

$64 - 60 = d^2$

$d^2 = 4$

∴ $d = \pm 2$

∴ the terms are: 8 – 2, 8, 8 + 2

Or 8 –(-2) , 8, 8 + (-2)

∴ the terms are: 6, 8, 10 OR 10 ,8, 6

7: A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5cm, 1cm, 1.5cm, 2cm..... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?

$\left[\text{Take } \pi = \frac{22}{7}\right]$

Sol: l_1, l_2, l_3 . . . be the length of semicircles of radius $r_1 = 0.5cm, r_2 = 1cm, r_3 = 1.5cm$ . . . respectively

Length of the semicirlce = πr

$\Rightarrow l_1 = \pi r_1 \Rightarrow 0.5\pi \Rightarrow \frac{\pi}{2}$

$l_2 = \pi, \Rightarrow 2\left(\frac{\pi}{2}\right),\ \ l_3 = 3\left(\frac{\pi}{2}\right) \ldots\ldots l_{13} = 13\left(\frac{\pi}{2}\right)$

Total length of the spiral = $l_1 + l_2 + l_3 + \ldots\ldots + l_{13}$

$\Rightarrow \frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) + 3\left(\frac{\pi}{2}\right) \ldots\ldots + 13\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{\pi}{2}(1 + 2 + 3 + \ldots\ldots + 13)$

$\Rightarrow \frac{\pi}{2}[\sum 13]$

$\Rightarrow \frac{\pi}{2}\left[\frac{13(13+1)}{2}\right]$

$\Rightarrow \frac{\pi}{2}\left[\frac{13x14}{2}\right] = \frac{1}{2} x \frac{22}{7} x13x7 = 11x13 = 143$

Exercise 2.3

1. If $T_n = 2n + 3$ find S_2

$S_2 = T_1 + T_2$

$S_2 = T_1 + T_2$

$T_1 = 2x1 + 3 \ \ = 2 + 3 = 5$

$T_2 = 2x2 + 3 \ = 4 + 3 = 7$

∴ $S_2 = 5 + 7$

∴ $S_2 = 12$

2. Find the sum of.

(i) 3+7+11+ . . . . . to 25 terms

a = 3, d = 4, n =25

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{25} = \frac{25}{2}[2x3 + (25-1)4]$

$S_{25} = \frac{25}{2}[6 + 24x4]$

$S_{25} = \frac{25}{2}[6 + 96]$

$S_{25} = \frac{25}{2}[102]$

$S_{25} = 25[51]$

$S_{25} = 1275$

(ii) -3, 1, 5, ... to 17 terms.

a = -3, d = 4, n =17

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{17}{2}[2.x(-3) + (17-1)4]$

$S_n = \frac{17}{2}[-6 + 16x4]$

$S_n = \frac{17}{2}[-6 + 64]$

$S_n = \frac{17}{2}[58]$

$S_n = 17[29]$

$S_n = 493$

(iii) 3a, a, -a, ………. to a terms

a = 3a, d = -2a, n =a

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{a}{2}[2(3a) + (a - 1)(-2a)]$

$S_n = \frac{a}{2}[6a + (-2a^2 + 2a)]$

$S_n = \frac{a}{2}[8a - 2a^2]$

$S_n = \frac{2a}{2}[4a - a^3]$

$S_n = a[4a - a^3]$

$S_n = 4a^2 - a^3$

(iv) p, o, -p, …….. to p terms

a = p, d = -p, n =p

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{p}{2}[2p + (p - 1)(-p)]$

$S_n = \frac{p}{2}[2p + (-p^2 + p)]$

$S_n = \frac{p^2}{2}[3 - p]$

3. Find the sum of the first 111 terms of an A.P. whose 56th term is $\frac{5}{37}$.

n = 111, $T_{56} = \frac{5}{37}$,

$T_n = a + (n – 1)d$

$a + (56 – 1)d = \frac{5}{37}$

$a + 55d = \frac{5}{37}$ ---------(1)

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{111} = \frac{111}{2}[2a + (111 - 1)d]$

$S_{111} = \frac{111}{2}[2a + 110d]$

$S_{111} = \frac{111x2}{2}[a + 55d]$

$S_{111} = 111\left[\frac{5}{37}\right]$ [Substitute in (i)]

$S_{111} = \frac{555}{37}$

$S_{111} = 15$

4. For a sequence of natural numbers,

(a) find (i) $\sum_{n=1}^{20} 20$ (ii) $S_{50} – S_{40}$ (iii) $S_{30} + S_{15}$

(b) find n' if (i) $S_n = 55$ (ii) $S_n = 15$

(a) (i) $\sum_{n=1}^{20} 20$

$\sum n = \frac{n(n+1)}{2}$

$\sum 20 = \frac{20(20+1)}{2}$

$\sum 20 = 10x21$
$\sum 20 = 210$

(ii) $S_{50} - S_{40}$
$S_n = \frac{n(n+1)}{2}$
$S_{50} - S_{40} = \frac{50(50+1)}{2} - \frac{40(40+1)}{2}$
$S_{50} - S_{40} = 25x51 - 20x41$
$S_{50} - S_{40} = 1275 - 820$
$S_{50} - S_{40} = 455$

(iii) $S_{30} + S_{15}$
$S_n = \frac{n(n+1)}{2}$
$S_{30} + S_{15} = \frac{30(30+1)}{2} + \frac{15(15+1)}{2}$
$S_{30} + S_{15} = \frac{30x31}{2} + \frac{15x16}{2}$
$S_{30} + S_{15} = 15x31 + 15x8$
$S_{30} + S_{15} = 465 + 120$
$S_{30} + S_{15} = 585$

(b) (i) $S_n = 55$
$S_n = \frac{n(n+1)}{2}$
$55 = \frac{n(n+1)}{2}$
$n(n+1) = 55x2$
$n(n+1) = 110$
$n(n+1) = 10(10+1)$
$\therefore n = 10$

(ii) $S_n = 15$
$S_n = \frac{n(n+1)}{2}$
$15 = \frac{n(n+1)}{2}$
$n(n+1) = 15x2$
$n(n+1) = 30$
$n(n+1) = 5(5+1)$
$\therefore n = 5$

5. Find the sum of all the first 'n' odd natural numbers.
$S_n = \frac{n}{2}[2a + (n-1)d]$
$a = 1, d = 2$
$\Rightarrow S_n = \frac{n}{2}[2x1 + (n-1)2]$
$\Rightarrow S_n = \frac{n}{2}[2 + 2n - 2]$
$\Rightarrow S_n = \frac{n}{2}[2n]$
$\Rightarrow S_n = n^2$

6. Find the sum of all natural numbers between 1 and 201 which are divisible by 5.
$=5 + 10 + 15 + 20 + ...+200$
$=5(1 + 2 + 3 + 4 + ...+40)$
$=5(S_n)$
$\Rightarrow S_n = \frac{n(n+1)}{2}$
$=5\left[\frac{40(40+1)}{2}\right]$

=5[20x41]
=5[820]
= 4100

7. Find the first four terms of a sequence of which sum to n terms is n $\frac{1}{2}$n(7n-1).

$S_n = \frac{1}{2}n(7n-1)$
$T_1 = S_1$
$T_1 = \frac{1}{2}x1(7x1-1)$
$T_1 = \frac{1}{2}(6)$
$T_1 = 3$
$T_1 + T_2 = S_2$
$\Rightarrow T_2 = S_2 - S_1$
$\Rightarrow T_2 = \frac{1}{2}x2(7x2-1) - 3$
$\Rightarrow T_2 = 1(14-1) - 3$
$\Rightarrow T_2 = 13 - 3$
$\Rightarrow T_2 = 10$
$T_3 = S_3 - S_2$
$\Rightarrow T_3 = \frac{1}{2}x3(7x3-1) - 13$
$\Rightarrow T_3 = \frac{3}{2}(21-1) - 13$
$\Rightarrow T_3 = \frac{3}{2}x20 - 13$
$\Rightarrow T_3 = 3x10 - 13 \Rightarrow 30-13$
$T_3 = 17$
$T_4 = S_4 - S_3$
$\Rightarrow T_4 = \frac{1}{2}x4(7x4-1) - 30$
$\Rightarrow T_4 = 2(28-1) - 30$
$\Rightarrow T_4 = 2(27) - 30$
$\Rightarrow T_4 = 54 - 30$
$\Rightarrow T_4 = 24$

8. How many terms of the A.P 1, 4, 7, ... are needed to make the sum 51?

$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = 51$, a = 1, d = 3
$\frac{n}{2}[2x1 + (n-1)3] = 51$
$\frac{n}{2}[2 + 3n - 3] = 51$
$\frac{n}{2}[3n - 1] = 51$
$n[3n - 1] = 102$
$3n^2 - n = 102$
$3n^2 - n - 102 = 0$
$3n^2 - 18n + 17n - 102 = 0$
$3n(n-6) + 17(n-6) = 0$
$(n-6) + (3n+17) = 0$
$(n-6) = 0 \Rightarrow n=6$

9. Find three numbers in AP whose sum and products are respectively.
(i) 21 and 231 (ii) 36 and.
The terms of an A.P. are – a – d, a, a + d
(i) a – d + a + a + d = 21

$3a = 21$
$a = 7$
$(a - d)\, a\, (a + d) = 231$
$(7 - d)\, 7\, (7 + d) = 231$
$(7 - d)(7 + d) = 33$
$7^2 - d^2 = 33$
$49 - d^2 = 33$
$-d^2 = 33 - 49$
$-d^2 = -16$
$d^2 = 16$
$d = \pm 4$
∴ The terms are - $7 - 4 = 3, 7, 7 + 4 = 11$
∴ The terms are 3 , 7, 11

(ii) $a - d + a + a + d = 36$
$3a = 36$
$a = 12$
$(a - d)\, a\, (a + d) = 1620$
$(12 - d)\, 12\, (12 + d) = 1620$
$(12 - d)(12 + d) = 135$
$12^2 - d^2 = 135$
$144 - d^2 = 135$
$-d^2 = 135 - 144$
$-d^2 = -9$
$d^2 = 9$
$d = \pm 3$
∴ The numbers are - $12 - 3 = 9, 12, 12 + 3 = 15$
∴ The numbers are 9 , 12, 15

10. The sum of 6 terms which form an A.P is 345. The difference between the first and last terms is 55. Find the terms.

$\frac{n}{2}[2a + (n - 1)d] = S_n$
$\frac{6}{2}[2a + (6 - 1)d] = 345$
$3[2a + 5d] = 345$
$6a + 15d = 345$
$2a + 5d = 115$-----------(1)
$T_n - a = 55$
$a + (n - 1)d - a = T_n$
$a + (6 - 1)d - a = 55$
$5d = 55$
$d = 11$
(i)$\Rightarrow 2a + 5x11 = 115$
$\Rightarrow 2a + 55 = 115$
$\Rightarrow 2a = 115 - 55$
$\Rightarrow 2a = 60$
$\Rightarrow a = 30$
∴ ⇒ The terms of an A.P. – 30, 41, 52, 63, 74, 85

11. In an AP whose first term is 2, the sum of first five terms is one fourth the sum of the next five terms. Show that $T_{20} = -112$ find S_{20}

$a = 2,\ S_5 = \frac{1}{4}(S_{10} - S_5)$

$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_5 = \frac{5}{2}[2x2 + (5-1)d]$
$S_5 = \frac{5}{2}[4 + 4d]$
$S_5 = 5[2 + 2d]$
$S_5 = 10 + 10d$
$S_{10} = \frac{10}{2}[2x2 + (10-1)d]$
$S_{10} = 5[4 + 9d]$
$S_{10} = 20 + 45d$
$S_{10} - S_5 = 20 + 45d - (10 + 10d)$
$S_{10} - S_5 = 20 + 45d - 10 - 10d$
$S_{10} - S_5 = 10 + 35d$
$10 + 10d = \frac{1}{4}(10 + 35d)$
$40 + 40d = 10 + 35d$
$5d = -30$
$d = -6$
$\therefore T_{20} = 2 + (20-1)-6$
$\therefore T_{20} = 2 + 19x-6$
$\therefore T_{20} = 2 - 114$
$\therefore T_{20} = -112$
$S_{20} = \frac{20}{2}[2x2 + (20-1)(-6)]$
$S_{20} = 10[4 - 114]$
$S_{20} = 10[-110]$
$S_{20} = -1100$

12. The third term of an A.P is 8 and the ninth term of the A.P exceeds three times the third term by 2. Find the sum of its first 19 terms.

$T_3 = 8$
$a + (3-1)d = 8$
$a + 2d = 8$---------(1)
$T_9 = 3T_3 + 2$
$a + (9-1)d = 3x8 + 2$
$a + 8d = 24 + 2$
$a + 8d = 26$-------(2)
(2) – (1)
$a + 8d = 26$
$a + 2d = 8$
$6d = 18$
$d = 3$
$\therefore$ (1) $\Rightarrow a + 2x\,3 = 8$
$\Rightarrow a + 6 = 8$
$\Rightarrow a = 2$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = \frac{19}{2}[2x2 + (19-1)3]$
$S_n = \frac{19}{2}[4 + 18x3]$
$S_n = \frac{19}{2}[4 + 54]$

$S_n = \frac{19}{2}[58]$
$S_n = 19x29$
$S_n = 551$

ILLUSTRATIVE EXAMPLES

1: In the H.P. $\frac{1}{5}, \frac{1}{3}$, 1, -1 - - - - find T_{10}.

$T_{10} = \frac{1}{a+(n-1)d}$
$T_{10} = \frac{1}{5+(10-1)(-2)}$
$T_{10} = \frac{1}{5+9(-2)}$
$T_{10} = \frac{1}{5-18}$
$T_{10} = -\frac{1}{13}$

2: In a H.P. $T_3 = \frac{1}{7}$ and $T_7 = \frac{1}{5}$. Find T_{15}.

In H.P. $T_3 = \frac{1}{7}$, $T_7 = \frac{1}{5}$
∴ The corresponding terms in A.P. are $T_3 = 7$, $T_7 = 5$
$d = \frac{T_p - T_q}{p-q}$
$d = \frac{5-7}{7-3}$
$d = \frac{-2}{4} \Rightarrow d = \frac{-1}{2}$
$T_{15} = T_7 + 8d$
$T_{15} = 5 + 8x\frac{-1}{2}$
$T_{15} = 5 - 4$
$T_{15} = 1$
∴ T_{15} of the H.P. $T_{15} = 1$

Exercise 2.4

1. Which of the following are harmonic progressions?
   (i) $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}$ . . . (ii) $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}$ . . . (iii) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{18}$ . . .
   (iv) $1, \frac{1}{3}, \frac{1}{7}, \frac{1}{11}$ . . . (v) 6, 4, 3, . . . (vi) $1, \frac{1}{2}, \frac{1}{4}$, . . .

(i) $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}$ . . .
The reciprocals are 1, 4, 7,10, . . .
4 -1 = 3, 7 – 4 = 3
The reciprocals are in A.P..
∴ Harmonic progressions

(ii) $1, \frac{2}{3}, \frac{1}{2}, \frac{2}{5}$ . . .
The reciprocals are $1, \frac{3}{2}, 2, \frac{5}{2}$ . .
$\frac{3}{2} - 1 = \frac{1}{2}, 2 - \frac{3}{2} = \frac{1}{2}, \frac{5}{2} - 2 = \frac{1}{2}$ . .
The reciprocals are in A.P
∴ Harmonic progressions

(iii) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{18}$ . . .
The reciprocals are 1, 2, 6, 18 . . .
2 – 1 =1, 6 – 2 =4 . . .

The reciprocals are not in A.P
∴ It is not a Harmonic progressions

(iv) $\frac{1}{3}, \frac{1}{7}, \frac{1}{11} \ldots$
The reciprocals are 3, 7, 11 . . .
7 – 3 = 4, 11 – 7 = 4
The reciprocals are in A.P.
∴ Harmonic progressions

(v) 6, 4, 3, . . .
The reciprocals are $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$
$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}, \frac{1}{3} - \frac{1}{4} = \frac{1}{12}, \ldots.$
The reciprocals are in A.P.
∴ Harmonic progressions

(vi) $1, \frac{1}{2}, \frac{1}{4}, \ldots$
The reciprocals are 1, 2, 4, . . .
$2 - 1 = 1, 4 - 1 = 3, \ldots.$
The reciprocals are not in A.P
∴ It is not a Harmonic progressions

2. Find
(i) T_n in $\frac{1}{2}, \frac{1}{4}, \frac{1}{6} \ldots T_n$ (ii) T_{10} in $\frac{1}{7}, \frac{1}{4}, 1 \ldots T_{10}$

(i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6} \ldots$ರಲ್ಲಿ T_n
$T_n = \frac{1}{a+(n-1)d}$
$T_n = \frac{1}{2+(n-1)2}$
$T_n = \frac{1}{2+2n-2}$
$T_n = \frac{1}{2n}$

(ii) $\frac{1}{7}, \frac{1}{4}, 1 \ldots$ರಲ್ಲಿ T_{10}
$T_n = \frac{1}{a+(n-1)d}$
$T_{10} = \frac{1}{7+(10-1)(-3)}$
$T_{10} = \frac{1}{7+9(-3)}$
$T_{10} = \frac{1}{7-27}$
$T_{10} = \frac{-1}{20}$

3. In a H.P. $T_5 = \frac{1}{12}$ and $T_{11} = \frac{1}{15}$, find T_{25}.
In H.P. $T_5 = \frac{1}{12}$
⇒In A.P. $T_5 = 12 = T_q$
In H.P. $T_{11} = \frac{1}{15}$
⇒ In A.P.$T_{11} = 15 = T_p$
$d = \frac{T_p - T_q}{p-q}$
$d = \frac{15-12}{11-5}$
$d = \frac{3}{6}$

d $= \frac{1}{2}$

$T_n = a + (n-1)d$

$12 = a + (5-1)\frac{1}{2}$

$12 = a + 4 \times \frac{1}{2}$

$12 = a + 2$

$a = 12 - 2$

$a = 10$

$\therefore T_{25} = 10 + (25-1)\frac{1}{2}$

$T_{25} = 10 + 24 \times \frac{1}{2}$

$T_{25} = 10 + 12$

$T_{25} = 22$

$\therefore$ In H.P. $T_{25} = \frac{1}{22}$

4. In a H.P. $T_4 = \frac{1}{11}$ and $T_{14} = \frac{3}{23}$, Find (i) T_7 (ii) T_{19}

In H.P. $T_4 = \frac{1}{11}$

$\Rightarrow$In A.P. $T_4 = 11 = T_q$

In H.P. $T_{14} = \frac{3}{23}$

$\Rightarrow$In A.P. $T_{14} = \frac{23}{3} = T_p$

d $= \frac{T_p - T_q}{p-q}$

d $= \frac{\frac{23}{3} - 11}{14-4}$

d $= \frac{-\frac{-10}{3}}{10}$

d $= -\frac{1}{3}$

$T_n = a + (n-1)d$

$11 = a + (4-1)(-\frac{1}{3})$

$11 = a + 3 \times (-\frac{1}{3})$

$11 = a - 1$

$a = 11 + 1$

$a = 12$

(i) $T_7 = 12 + (7-1)(-\frac{1}{3})$

$T_7 = 12 + 6 \times (-\frac{1}{3})$

$T_7 = 12 - 2$

$T_7 = 10$

$\Rightarrow$In an A.P. $T_7 = 10$

$\therefore$ In H.P. $T_7 = \frac{1}{10}$

(ii) $T_{19} = 12 + (19-1)(-\frac{1}{3})$

$T_{19} = 12 + 18 \times (-\frac{1}{3})$

$T_{19} = 12 - 6$

$T_{19} = 6$

$\Rightarrow$In A.P. $T_{19} = 6$

$\therefore$ In H.P. $T_{19} = \frac{1}{6}$

ILLUSTRATIVE EXAMPLES

1: Find the first 3 terms of a G.P if a = 4 and r = 2.

$T_1 = a = 4$

$T_2 = ar = 4x2 = 8$

$T_3 = ar^2 = 4x2^2 = 4x4 = 16$

∴The three terms are: 4, 8, 16

2:Find the fifth term of the G.P $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

$a = \frac{3}{2}$, $r = \frac{1}{2}$,

$T_n = ar^{n-1}$

$T_5 = \frac{3}{2} x \left(\frac{1}{2}\right)^{5-1}$

$T_5 = \frac{3}{2} x \left(\frac{1}{2}\right)^{4}$

$T_5 = \frac{3}{2} x \frac{1}{2^4}$

$T_5 = \frac{3}{2} x \frac{1}{16}$

$T_5 = \frac{3}{32}$

3: Which term of the G. P $2, 2\sqrt{2}, 4, \ldots$ is 32?

$T_n = ar^{n-1}$

$32 = 2 x \left(\sqrt{2}\right)^{n-1}$

$16 = \left(2^{\frac{1}{2}}\right)^{n-1}$

$2^4 = 2^{\frac{n-1}{2}}$

$4 = \frac{n-1}{2}$

$8 = n - 1$

$n = 8 + 1 = 9$

5: The first term of the G.P is 25 and 6^{th} term is 800. Find the seventh term.

$T_n = ar^{n-1}$

$800 = 25 x r^5$

$r^5 = 32$

$r^5 = 2^5$

∴ r = 2

$T_7 = r x T_6$

$T_7 = 2 x 800$

$T_7 = 1600$

Exercise 2.5

1. Find the common ratio in the following G.P.

(i) $-5, 1, \frac{-1}{5}, \ldots$ (ii) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$

(i) $-5, 1, \frac{-1}{5}, \ldots$

$r = \frac{1}{-5}$,

(ii) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$

$r = \frac{3}{\sqrt{3},} = \frac{3\sqrt{3}}{(\sqrt{3})^2} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

2. Do as directed.

(i) If a = 1 and $r = \frac{2}{3}$ find (a) T_n (b) T_4

(ii) In the G.P. 729,243, 81, . . .find T_7

(i) (a) $T_n = a.r^{n-1}$

$T_n = 1.\left(\frac{2}{3}\right)^{n-1}$

$T_n = \left(\frac{2}{3}\right)^{n-1}$

(b) $T_4 = \left(\frac{2}{3}\right)^{4-1}$

$T_4 = \left(\frac{2}{3}\right)^{3}$

$T_4 = \frac{2^3}{3^3}$

$T_4 = \frac{8}{27}$

(ii) In the G.P. 729,243, 81, . . .find T_7

$T_n = a.r^{n-1}$

$a = 729$, $r = \frac{243}{729} = \frac{1}{3}$

$T_7 = 729.\left(\frac{1}{3}\right)^{7-1}$

$T_7 = 729.\left(\frac{1}{3}\right)^{6}$

$T_7 = 729.\frac{1^6}{3^6}$

$T_7 = 729.\frac{1}{729}$

$T_7 = 1$

3. Find the 12th term of a G.P whose 5th term is 64 and common ratio is 2.

$T_5 = 64$, $r = 2$, $T_{12} = ?$

$T_n = a.r^{n-1}$

$T_5 = a\,(2)^{5-1}$

$64 = a\,(2)^4$

$64 = 16a$

$a = \frac{64}{16}$

$a = 4$

$T_{12} = 4 \times 2^{12-1}$

$T_{12} = 4 \times 2^{11}$

$T_{12} = 4 \times 2048$

$T_{12} = 8192$

4. Find the following.
   (i) 10th and 16th terms of the G.P. 256, 128, 64, . . .
   (ii) 8th and 12th terms of the G.P. 81, -27, 9, . . .
   (iii)4th and 8th terms of the G.P. 0.008, 0.04, 0.2 .....

(i) 10th and 16th terms of the G.P. 256, 128, 64, . . .

$a = 729$, $r = \frac{128}{256} = \frac{1}{2}$

$T_n = a.r^{n-1}$

$T_{10} = 256 \times \left(\frac{1}{2}\right)^{10-1}$

$T_{10} = 256 \times \left(\frac{1}{2}\right)^{9}$

$T_{10} = 256 \times \frac{1^9}{2^9}$

$T_{10} = 256 \times \frac{1}{512}$

$T_{10} = \frac{1}{2}$

$T_{16} = 256 \text{ x } \left(\frac{1}{2}\right)^{16-1}$

$T_{16} = 256 \text{ x } \left(\frac{1}{2}\right)^{15}$

$T_{16} = 256 \text{ x } \frac{1}{2^{15}}$

$T_{16} = 256 \text{ x } \frac{1}{256\text{x}2^7}$

$T_{16} = \frac{1}{128}$

Or

$T_{16} = T_{10} \text{ x } r^6$

$T_{16} = \frac{1}{2} \text{ x } \left(\frac{1}{2}\right)^6$

$T_{16} = \frac{1}{2^7}$

$T_{16} = \frac{1}{128}$

(ii) 8th and 12th terms of the G.P. 81, -27, 9, . . .

a = 81,

$r = \frac{-27}{81} = \frac{-1}{3}$

$T_8 = 81.\left(\frac{-1}{3}\right)^{8-1}$

$T_8 = 81.\left(\frac{-1}{3}\right)^{7}$

$T_8 = 81.\frac{-1}{3^7}$

$T_8 = 3^4.\frac{-1}{3^7}$

$T_8 = \frac{-1}{3^3}$

$T_8 = \frac{-1}{27}$

$T_{12} = 81.\left(\frac{-1}{3}\right)^{12-1}$

$T_8 = 81.\left(\frac{-1}{3}\right)^{11}$

$T_8 = 81.\frac{-1}{3^{11}}$

$T_8 = 3^4.\frac{-1}{3^{11}}$

$T_8 = \frac{-1}{3^7}$

$T_8 = \frac{-1}{2187}$

or

$T_{12} = T_8 \text{x} r^4$

$T_{12} = \frac{-1}{27}\left(\frac{-1}{3}\right)^4$

$T_{12} = \frac{-1}{27} \text{ x } \frac{1}{81}$

$T_{12} = \frac{-1}{2187}$

(iii) 4th and 8th terms of the G.P. 0.008, 0.04, 0.2 .....

$a = 0.008 = \frac{8}{1000} = \left(\frac{1}{5}\right)^3$, $r = \frac{0.04}{0.008} = \frac{40}{8} = 5$

$T_n = a.r^{n-1}$

$T_4 = \left(\frac{1}{5}\right)^3 \times 5^{4-1}$

$T_4 = \frac{1}{5^3} \times 5^3$

$T_4 = 1$

$T_8 = \left(\frac{1}{5}\right)^3 . 5^7$

$T_8 = \frac{1}{5^3} \times 5^7$

$T_8 = 5^4$

$T_8 = 625$

or

$T_8 = T_4 . r^4$

$T_8 = 1 . 5^4$

$T_8 = 625$

5. Find the last term of the following sequence:.

(i) 2, 4, 8, . . . to 9 terms (ii) 4, 4^2, 4^3 . . . to 2n terms

(iii) 2, 3, $4\frac{1}{2}$, . . . to 6 terms (iv) x, 1, $\frac{1}{x}$, . . . to 30 terms

(i) 2, 4, 8, . . . to 9 terms

$a = 2, r = \frac{4}{2} = 2$

$T_n = a.r^{n-1}$

$T_9 = 2.2^{9-1}$

$T_9 = 2^9$

$T_9 = 512$

(ii) 4, 4^2, 4^3 . . . to 2n terms

$a = 4, r = \frac{4^2}{4} = 4$

$T_{2n} = 4.4^{2n-1}$

$T_{2n} = 4^{2n}$

(iii) 2, 3, $4\frac{1}{2}$, . . . to 6 terms

$a = 2, r = \frac{3}{2}$

$T_n = a.r^{n-1}$

$T_6 = 2. \left(\frac{3}{2}\right)^{6-1}$

$T_6 = 2. \left(\frac{3}{2}\right)^{5}$

$T_6 = 2.\frac{3^5}{2^5}$

$T_6 = \frac{2x243}{32}$

$T_6 = \frac{243}{16}$

(iv) x, 1, $\frac{1}{x}$, . . . to 30 terms

$a = x, r = \frac{1}{x}$

$T_n = a.r^{n-1}$

$T_{30} = x. \left(\frac{1}{x}\right)^{30-1}$

$T_{30} = x. \left(\frac{1}{x}\right)^{29}$

$T_{30} = \frac{x}{x^{29}}$

$T_{30} = \frac{1}{x^{28}}$

6. Find the G.P if $T_5 : T_{10} = 32 : 1$ and $T_7 = \frac{1}{32}$.

$T_5 : T_{10} = 32 : 1$

$\frac{T_{10}}{T_5} = \frac{1}{32}$

$\frac{a.r^{10-1}}{a.r^{5-1}} = \frac{1}{32}$

$\frac{r^9}{r^4} = \frac{1}{32}$

$r^5 = \frac{1}{2^5}$

$r = \frac{1}{2}$

$T_7 = \frac{1}{32}$

$a.\left(\frac{1}{2}\right)^{7-1} = \frac{1}{32}$

$a.\frac{1}{2^6} = \frac{1}{32}$

$a = \frac{64}{32}$

$a = 2$

$a = 2$, $T_2 = a.r = 1$; $T_3 = T_2.r = \frac{1}{2}$; $T_4 = T_3.r = \frac{1}{4}$; $T_5 = \frac{1}{8}$; $T_6 = \frac{1}{16}$

∴ ಗುಣೋತ್ತರ ಶ್ರೇಢಿ 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

7. The half life period of a certain radioactive material is 1 hour. If the initial sample weighed 500 gm, find the mass of the sample remaining at the end of 5^{th} hour?

$a = 500$, $r = \frac{1}{2}$, $n = 6$ [After 5 hours number of terms are to be 6 including first term]

$T_n = a.r^{n-1}$

$T_5 = 500.\left(\frac{1}{2}\right)^{6-1}$

$T_5 = 500.\left(\frac{1}{2}\right)^{5}$

$T_5 = 500.\frac{1}{2^5}$

$T_5 = 500.\frac{1}{32}$

$T_5 = \frac{500}{32}$

$T_5 = 15$ Gram

8. Which term of the sequence 3, 6, 12, . . . is 1536?

$a = 3$, $r = \frac{6}{3} = 2$, $T_n = 1536$

$a.r^{n-1} = T_n$

$3.2^{n-1} = 1536$

$2^{n-1} = \frac{1536}{3}$

$2^{n-1} = 512$

$2^{n-1} = 2^9$

$n - 1 = 9$

$n = 10$

9. If the 4^{th} and 8^{th} terms of a GP are 24 and 384 respectively, find the first term and common ratio.

$T_4 = 24$; $T_8 = 384 \frac{T_8}{T_4} = \frac{384}{24}$

$\frac{a.r^{8-1}}{a.r^{4-1}} = 16$

$\frac{r^7}{r^3} = 16$

$r^4 = 2^4$

$r = 2$

$a.r^{n-1} = T_n$

$a.2^{4-1} = 24$

$a.2^3 = 24$

$a = \frac{24}{8}$

$a = 3$

10. Find the G.P. in which.

(i) The 10^{th} term is 320 and 6^{th} term is 20

(ii) 2^{nd} term is $\sqrt{6}$ and 6^{th} term is $9\sqrt{6}$

(i) The 10^{th} term is 320 and 6^{th} term is 20

$T_{10} = 320$; $T_6 = 20$

$\frac{T_{10}}{T_6} = \frac{320}{20}$

$\frac{a.r^{10-1}}{a.r^{6-1}} = 16$

$\frac{r^9}{r^5} = 16$

$r^4 = 2^4$

$r = 2$

$a.r^{n-1} = T_n$

$a.2^{6-1} = 20$

$a.2^5 = 20$

$a = \frac{20}{32}$

$a = \frac{5}{8}$

The G.P. is - $\frac{5}{8}$, $\frac{5}{8}$x2 $= \frac{5}{4}$, $\frac{5}{4}$x2 $= \frac{5}{2}$ . . .

$\frac{5}{8}$, $\frac{5}{4}$, $\frac{5}{2}$, . . .

(ii) 2^{nd} term is $\sqrt{6}$ and 6^{th} term is $9\sqrt{6}$

$T_6 = 9\sqrt{6}$; $T_2 = \sqrt{6}$

$\frac{T_6}{T_2} = \frac{9\sqrt{6}}{\sqrt{6}}$

$\frac{a.r^{6-1}}{a.r^{2-1}} = 9$

$\frac{r^5}{r} = 9$

$r^4 = 3^2$

$r^2 = 3$

$r = \sqrt{3}$

$a.r^{n-1} = T_n$

$a.(\sqrt{3})^{2-1} = \sqrt{6}$

$a.\sqrt{3} = \sqrt{6}$

$a = \frac{\sqrt{6}}{\sqrt{3}}$

$a = \sqrt{2}$

The G.P. is - $\sqrt{2}$, $\sqrt{2}$x$\sqrt{3}$ =$\sqrt{6}$, $\sqrt{6}$x$\sqrt{3}$ = $\sqrt{12}$ = $2\sqrt{3}$. . .

$\sqrt{2}$, $\sqrt{6}$, $2\sqrt{3}$, . . .

ILLUSTRATIVE EXAMPLES

1: Find the sum of first 6 terms of the G.P 3, 6, 12, ...

a = 3, r = 2, n = 6

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$S_6 = 3\left(\frac{2^6-1}{2-1}\right)$

$S_6 = 3\left(\frac{64-1}{1}\right)$

$S_6 = 3\left(\frac{63}{1}\right) = 3 \times 63$

$S_6 = 189$

2: How many terms of the series 1 + 4 + 16 + ... make the sum 1,365?

a = 1, r = 4, n = ? $S_n = 1365$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$1365 = 1\left(\frac{4^n-1}{4-1}\right)$

$1365 = \frac{4^n-1}{3}$

$4095 = 4^n - 1$

$4^n = 4096$

$4^n = 4^6$

$\therefore n = 6$

3: Find the sum to infinity of the geometric series $2 + \frac{2}{3} + \frac{2}{9} + \ldots..$

a = 2, $r = \frac{1}{3}$, n = ? $S_\infty = ?$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{2}{1-\frac{1}{3}}$

$S_\infty = \frac{2}{\frac{3-1}{3}}$

$S_\infty = \frac{2}{\frac{2}{3}} \Rightarrow S_\infty = 2 \times \frac{3}{2} = 3$

4: If the third term of a G.P is 12 and its sixth term is 96, find the sum of 9 terms

$T_3 = 12, T_6 = 96$

$\frac{T_3}{T_6} = \frac{12}{96}$

$\frac{ar^2}{ar^5} = \frac{12}{96}$

$\frac{r^2}{r^5} = \frac{1}{8}$

$\frac{1}{r^3} = \frac{1}{2^3}$

$r^3 = 2^3$

$r = 2$

$T_3 = 12$

$a \times 2^2 = 12$

$4a = 12$

$a = 3$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$S_9 = 3\left(\frac{2^9-1}{2-1}\right)$

$S_9 = 3\left(\frac{512-1}{1}\right)$

$S_9 = 3(511)$

$S_9 = 1533$

5: Sum of three terms in a G.P is 31 and their product is 125. Find the numbers.

$\frac{a}{r}$ x a x ar = 125

$a^3 = 125$

a = 5

$\frac{a}{r} + a + ar = 31$

$5 + 5r + 5r^2 = 31r$

$5r^2 - 26r + 5 = 0$

$5r^2 - 25r - r + 5 = 0$

$5r(r - 5) - 1(r - 5) = 0$

$(5r - 1)(r - 5) = 0$

$\Rightarrow 5r = 1$ or $r = 5$

$\Rightarrow r = \frac{1}{5}$ or $r = 5$

∴ The terms of G.P.: $\frac{5}{\frac{1}{5}}$, 5, $5\text{x}\frac{1}{5}$ Or $\frac{5}{5}$, 5, 5x5

∴ The terms of G.P.: 25 , 5, 1 Or 1 , 5, 25

Exercise 2.6

1. Find the sum of the following geometric series.
   (i) 1 + 2 + 3 + 4 +. . . up to 10 terms
   (ii) $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ up to ∞ terms

(i) 1 + 2 + 3 + 4 +. . . up to 10 terms

$a = 1, r = \frac{2}{1} = 2, n = 10$

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

$S_{10} = 1\left(\frac{2^{10} - 1}{2 - 1}\right)$

$S_{10} = 1\left(\frac{2^{10} - 1}{2 - 1}\right)$

$S_{10} = \left(\frac{1024 - 1}{1}\right)$

$S_{10} = 1023$

(ii) $1 + \frac{1}{3} + \frac{1}{9} + \ldots$ up to ∞

$a = 1, r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{3}{9} = \frac{1}{3}; \quad n = \infty$

$S_\infty = \frac{a}{1 - r}$

$S_\infty = \frac{1}{1 - \frac{1}{3}}$

$S_\infty = \frac{\frac{1}{2}}{3}$

$S_\infty = \frac{3}{2}$

Find the first term of a G.P in which S

2. Find the first term of a G.P in which $S_8 = 510$ and $r = 2$.

$a\left(\frac{r^n-1}{r-1}\right) = S_n$

$a\left(\frac{2^8-1}{2-1}\right) = 510$

$a\left(\frac{256-1}{1}\right) = 510$

$255a = 510$

$a = \frac{510}{255}$

$a = 2$

3. Find the sum.

(i) $1 + 2 + 4 + \; . \; . \; . \; + 512$

(ii) $\frac{1}{2} + \frac{1}{4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{2^{10}}$

(i) $1 + 2 + 4 + \; . \; . \; . \; + 512$

$a = 1, r = 2, T_n = 512$

$T_n = a.r^{n-1}$

$512 = 1.2^{n-1}$

$2^9 = 2^{n-1}$

$n - 1 = 9$

$n = 10$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$S_{10} = 1\left(\frac{2^{10}-1}{2-1}\right)$

$S_{10} = \left(\frac{1024-1}{1}\right)$

$S_{10} = 1023$

(ii) $\frac{1}{2} + \frac{1}{4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{2^{10}}$

$a = \frac{1}{2}$, $r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$, n=10

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

$S_{10} = \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}}\right)$

$S_{10} = \frac{1}{2}\left(\frac{1-\frac{1}{2^{10}}}{\frac{1}{2}}\right)$

$S_{10} = \frac{2}{1} \text{x} \frac{1}{2}\left(\frac{2^{10}-1}{2^{10}}\right)$

$S_{10} = 1\left(\frac{1024-1}{1024}\right)$

$S_{10} = \frac{1023}{1024}$

4. How many terms of the series $2 + 4 + 8 + \ldots\ldots$ make the sum 1022?

$a = 2, r = 2, S_n = 1022$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$

$2\left(\frac{2^n-1}{2-1}\right) = 1022$

$2\left(\frac{2^n-1}{1}\right) = 1022$

$2^{n+1} - 2 = 1022$

$2^{n+1} = 1024$

$2^{n+1} = 2^{10}$

$n + 1 = 10$
$n = 9$

5. Find.
   (i) $S_2 : S_4$ for the series 5 + 10 + 20 +……
   (ii) $S_4 : S_8$ for the series 4 + 12 + 36 +…….

(i) $S_2 : S_4$ for the series 5 + 10 + 20 +……

$\frac{S_2}{S_4} = \frac{a\left(\frac{r^2-1}{r-1}\right)}{a\left(\frac{r^4-1}{r-1}\right)}$

$\frac{S_2}{S_4} = \frac{r^2-1}{r^4-1}$

$\frac{S_2}{S_4} = \frac{r^2-1}{(r^2+1)(r^2-1)}$

$\frac{S_2}{S_4} = \frac{1}{(r^2+1)}$

$\frac{S_2}{S_4} = \frac{1}{(2^2+1)}$

$\frac{S_2}{S_4} = \frac{1}{4+1}$

$\frac{S_2}{S_4} = \frac{1}{5}$

$S_2 : S_4 = 1 : 5$

Alternate:

$S_n : S_{2n} = 1 : 1 + r^n$
$S_2 : S_4 = 1 : 1 + 2^2$
$S_2 : S_4 = 1 : 1 + 4$
$S_2 : S_4 = 1 : 5$

(ii) $S_4 : S_8$ for the series 4 + 12 + 36 +…….

$\frac{S_4}{S_8} = \frac{a\left(\frac{r^4-1}{r-1}\right)}{a\left(\frac{r^8-1}{r-1}\right)}$

$\frac{S_4}{S_8} = \frac{r^4-1}{r^8-1}$

$\frac{S_4}{S_8} = \frac{r^4-1}{(r^4+1)(r^4-1)}$

$\frac{S_4}{S_8} = \frac{1}{(r^4+1)}$

$\frac{S_4}{S_8} = \frac{1}{(3^4+1)}$

$\frac{S_4}{S_8} = \frac{1}{81+1}$

$\frac{S_4}{S_8} = \frac{1}{82}$

$S_4 : S_8 = 1 : 82$

Alternate:

$S_n : S_{2n} = 1 : 1 + r^n$
$S_4 : S_8 = 1 : 1 + 3^4$
$S_2 : S_4 = 1 : 1 + 81$
$S_2 : S_4 = 1 : 82$

6. Find the G.P if (i) $S_6 : S_3 = 126:1$ and $T_4 = 125$ (ii) $S_{10} : S_5 = 33:32$ and $T_5 = 64$

(i) $S_6 : S_3 = 126:1$ ಮತ್ತು $T_4 = 125$

$\frac{S_6}{S_3} = \frac{a\left(\frac{r^6-1}{r-1}\right)}{a\left(\frac{r^3-1}{r-1}\right)}$

$\frac{126}{1} = \frac{r^6-1}{r^3-1}$

$126 = \frac{(r^3+1)(r^3-1)}{(r^3-1)}$

$126 = (r^3 + 1)$

$r^3 = 126 - 1$

$r^3 = 125$

$r^3 = 5^3$

$r = 5$

$a.5^{4-1} = 125$

$a.5^3 = 125$

$a = \frac{125}{125}$

$a = 1$

∴ The G.P. is - 1, 5, 25, 125 . . . . ..

(ii) $S_{10} : S_5 = 33:32$ ಮತ್ತು $T_5 = 64$

$\frac{S_{10}}{S_5} = \frac{a\left(\frac{r^{10}-1}{r-1}\right)}{a\left(\frac{r^5-1}{r-1}\right)}$

$\frac{33}{32} = \frac{r^{10}-1}{r^5-1}$

$\frac{33}{32} = r^5 + 1$

$r^5 = \frac{33}{32} - 1$

$r^5 = \frac{33-32}{32}$

$r^5 = \frac{1}{32}$

$r^5 = \left(\frac{1}{2}\right)^5$

$r = \frac{1}{2}$

$a\left(\frac{1}{2}\right)^{5-1} = 64$

$a\left(\frac{1}{2}\right)^4 = 64$

$\frac{a}{16} = 64$

$a = 64x16$

$a = 1024$

∴ The G.P. is - 1024, 512, 256, 128, 64 . . . . ..

7. The first term of an infinite geometric series is 6 and its sum is 8. Find the G.P.

$a = 6, S_\infty = 8$

$S_\infty = \frac{a}{1-r}$

$8 = \frac{6}{1-r}$

$1 - r = \frac{6}{8}$

$r = 1 - \frac{3}{4}$

$r = \frac{1}{4}$

∴ The G.P. is - 6, $6x\frac{1}{4} = \frac{3}{2}, \frac{3}{2}x\frac{1}{4} = \frac{3}{8}$ . . .

∴ The G.P.is - 6, $\frac{3}{2}, \frac{3}{8}$ . . .

Find 3 terms in G.P whose sum and product respectively are

8. Find 3 terms in G.P whose sum and product respectively are
   (i) 7 and 8 (ii) 21 and 216 (iii) 19 and.

(i) 7 and 8

Let the three terms of G.P. are - $\frac{a}{r}$, a , ar

$\frac{a}{r}$ x a x ar = 8

$a^3 = 2^3$

a = 2 ------- (1)

$\frac{2}{r} + 2 + 2r = 7$

$2 + 2r + 2r^2 = 7r$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

2r(r -2) –1(r - 2) = 0

(r -2) (2r - 1) = 0

r =2 and r = $\frac{1}{2}$

∴ Three terms are – $\frac{2}{2}$, 2, 2x2 ⇒ 1, 2, 4 . . .or 4, 2, 1

(ii) 21 and 216

Let the three terms of G.P. are - $\frac{a}{r}$, a , ar

$\frac{a}{r}$ x a x ar = 216

$a^3 = 6^3$

a = 6 ------- (1)

$\frac{6}{r} + 6 + 6r = 21$

$6 + 6r + 6r^2 = 21r$

$6r^2 - 15r + 6 = 0$

$2r^2 - 5r + 2 = 0$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

2r(r -2) –1(r - 2) = 0

(r -2) (2r - 1) = 0

r =2 and r = $\frac{1}{2}$

∴ Three terms are – $\frac{6}{2}$, 6, 6x2 ⇒ 3, 6, 12 . . .or 12, 6, 3

(iii) 19 and 216

$\frac{a}{r}$ x a x ar = 216

$a^3 = 6^3$

a = 6 ------- (1)

$\frac{6}{r} + 6 + 6r = 19$

$6 + 6r + 6r^2 = 19r$

$6r^2 - 13r + 6 = 0$

$6r^2 - 9r - 4r + 6 = 0$

3r(2r -3)-2(2r – 3) = 0

(2r -3)(3r – 2) = 0

(2r -3) = 0 or (3r - 2) = 0

r = $\frac{3}{2}$ or r = $\frac{2}{3}$

∴ Three terms are – 6x$\frac{2}{3}$, 6, 6x$\frac{3}{2}$ ⇒ 4, 6, 9 . . .or 9, 6, 4

9. A person saved every year half as much he saved the previous year. If he totally saved Rs 19,375 in 5 years, how much did he save the first year?

a = ? , $r = \frac{1}{2}$, n = 5, S_5 = 19,375

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

$S_5 = a\left[\frac{1-\left(\frac{1}{2}\right)^5}{1-\frac{1}{2}}\right]$

$19{,}375 = a\left[\frac{1-\frac{1}{32}}{\frac{1}{2}}\right]$

$19{,}375 = 2a\left[\frac{32-1}{32}\right]$

$19{,}375 = a\left[\frac{31}{16}\right]$

16x 19,375 = 31a

$a = \frac{16x\ 19{,}375}{31}$

a = 10,000

ILLUSTRATIVE EXAMPLES

1: Find the arithmetic mean between 7 and 13.

Sol:: $A = \frac{a+b}{2}$

$A = \frac{7+13}{2}$

$A = \frac{20}{2}$

A = 10

2: Find 'x' if (p – q), x, (p + q) are in A.P.

Sol: $A = \frac{a+b}{2}$

$x = \frac{(p-q)+(p+q)}{2}$

$x = \frac{2p}{2} \Rightarrow x = p$

ILLUSTRAYIVE EXAMPLES

1: Find the GM between 4 and 36.

Sol:$G = \sqrt{ab}$

$G = \sqrt{4x36}$

$G = \sqrt{144}$

G = 12

2: Find x if 6, x + 2, 54 are in G.P.

Sol: $G = \sqrt{ab}$

$x+2 = \sqrt{6x54}$

$x+2 = \sqrt{324}$

x+2 = 18

x = 18 - 2

x = 16

Exercise 2.7

1. Find the AM, GM and HM between.

(i) 12 ಮತ್ತು 30 (ii) $\frac{1}{2}$ ಮತ್ತು $\frac{1}{8}$ (iii) -8 ಮತ್ತು -42 (iv) 9 ಮತ್ತು 18

(i) 12 and 30

A.M.: $A = \frac{a+b}{2} = \frac{12+30}{2} = \frac{42}{2} = 21$

G.M.: $G = \sqrt{ab} = \sqrt{12x30} = \sqrt{360} = 6\sqrt{10}$

H.M.: $H = \frac{2ab}{a+b} = \frac{2x12x30}{12+3o} = \frac{720}{42} = \frac{120}{7}$

(ii) $\frac{1}{2}$ and $\frac{1}{8}$

A.M.: $A = \frac{a+b}{2} = \frac{\frac{1}{2}+\frac{1}{8}}{2} = \frac{\frac{4+1}{8}}{2} = \frac{\frac{5}{8}}{2} = \frac{5}{16}$

G.M.: $G = \sqrt{ab} = \sqrt{\frac{1}{2}x\frac{1}{8}} = \sqrt{\frac{1}{16}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

H.M.: $H = \frac{2ab}{a+b} = \frac{2x\frac{1}{2}x\frac{1}{8}}{\frac{1}{2}+\frac{1}{8}} = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{8} x \frac{8}{5} = \frac{1}{5}$

(iii) -8 and -14

A.M.: $A = \frac{a+b}{2} = \frac{-8-42}{2} = \frac{-50}{2} = -25$

G.M.: $G = \sqrt{ab} = \sqrt{-8x(-42)} = \sqrt{336} = \sqrt{16x21} = 4\sqrt{21}$

H.P.: $H = \frac{2ab}{a+b} = \frac{2(-8)(-42)}{-8-42} = \frac{672}{-50} = \frac{-336}{25}$

(iv) 9 and 18

A.M.: $A = \frac{a+b}{2} = \frac{9+18}{2} = \frac{27}{2}$

G.M.: $G = \sqrt{ab} = \sqrt{9x18} = \sqrt{142} = \sqrt{81x2} = 9\sqrt{2}$

H.M.: $H = \frac{2ab}{a+b} = \frac{2(9)(18)}{9+18} = \frac{324}{27} = 12$

2. Find x, if 5, 8, x are in H.P.

$H = \frac{2ab}{a+b}$

$8 = \frac{2(5)(x)}{5+x} \Rightarrow 8 = \frac{10x}{5+x} \Rightarrow 8(5+x) = 10x$

$\Rightarrow 40 + 8x = 10x \Rightarrow 2x = 40 \Rightarrow x = 20$

3. Find x, if the following are in A.P.

(i) 5, (x-1), 0 (ii) $(a+b)^2$, x, $(a-b)^2$

(i) 5, (x-1), 0

$A = \frac{a+b}{2} \Rightarrow (x-1) = \frac{5+0}{2}$

$\Rightarrow x-1 = \frac{5}{2}$

$\Rightarrow x = \frac{5}{2} + 1$

$\Rightarrow x = \frac{5+2}{2} = \frac{7}{2}$

(ii) $(a+b)^2$, x, $(a-b)^2$

$A = \frac{a+b}{2}$

$x = \frac{(a+b)^2+(a-b)^2}{2}$

$x = \frac{a^2+b^2+2ab+a^2+b^2-2ab}{2}$

$x = \frac{2a^2+2b^2}{2}$

$x = a^2 + b^2$

4. The product of two numbers is 119 and their AM is 12. Find the numbers.

$ab = 119$, $\frac{a+b}{2} = 12$

$\frac{a+b}{2} = 12 \Rightarrow a + b = 24$

$\Rightarrow b = 24 - a$

$ab = 119$

$a(24 - a) = 119$
$24a - a^2 = 119$
$a^2 - 24a + 119 = 0$
$a^2 - 17a - 7a + 119 = 0$
$a(a - 17) - 7(a - 17) = 0$
$(a - 17)(a - 7) = 0$
$\Rightarrow a = 17$ and $a = 7$

5. Find x, if $\sqrt{2}$,x, $\frac{1}{\sqrt{2}}$ are in G.P.

$G = \sqrt{ab}$
$x = \sqrt{ab}$
$x^2 = ab$
$x^2 = \sqrt{2} \times \frac{1}{\sqrt{2}}$
$x^2 = 1$
$x = 1$

6. The arithmetic mean of two numbers is 17 and their geometric mean is 15. Find the numbers.

$A = \frac{a+b}{2} = 17$, $G = \sqrt{ab} = 15$
$\frac{a+b}{2} = 17$
$\Rightarrow a + b = 34$
$b = 34 - a$ -------(1)
$\sqrt{ab} = 15$
$\Rightarrow ab = 225$
$\Rightarrow a(34 - a) = 225$ [From (1)]
$\Rightarrow 34a - a^2 = 225$
$\Rightarrow a^2 - 34a + 225 = 0$
$\Rightarrow a^2 - 25a - 9a + 225 = 0$
$\Rightarrow a(a - 25) - 9(a - 25) = 0$
$\Rightarrow (a - 25)(a - 9) = 0$
$\Rightarrow a = 25$ ಅಥವಾ $a = 9$

$\therefore b = 34 - 25 = 9$ or $b = 34 - 9 = 25$
$\therefore$ Numbers are 9 and 25

7. The arithmetic mean of two numbers is $\frac{13}{2}$ and their geometric mean is 6.find their harmonic mean.

$A = \frac{a+b}{2} = \frac{13}{2}$, $G = \sqrt{ab} = 6$
$\frac{a+b}{2} = \frac{13}{2}$
$\Rightarrow a + b = 13$
$\sqrt{ab} = 6$
$\Rightarrow ab = 36$
$H = \frac{2ab}{a+b}$
$H = \frac{2 \times 36}{13}$
$H = \frac{72}{13}$